VIII. Fracture Mechanics

The field of fracture mechanics is highly developed and widely applied. There are many excellent sources of information on the subject and an independent treatment here is certainly not needed. There does however seem to be some confusion about the separate roles of fracture mechanics and failure criteria. This section considers some of the issues, and seeks a clarification of the separate and distinct capabilities of the two disciplines.

The terminology "failure criteria" is as introduced in Section I, as used in this entire website, and consistent with common usage. In jointly considering failure criteria and fracture mechanics some basic and obvious questions are as follows. Are failure criteria and fracture mechanics, as commonly practiced, completely independent fields? Alternatively, are they largely independent, but with some degree of overlap?

Another question is foremost in considering fracture mechanics and failure criteria. Is one approach more general and inherently superior to the other, as is sometimes said? This coordinated look at the two fields begins with a highly concentrated summary of the remarkable development of the field of fracture mechanics.

Fracture Mechanics Development

It all originated with Griffith in 1921. He recognized that flaws could induce failure in materials and he posed and solved the idealized problem of a single crack in an infinite two-dimensional, isotropic, elastic medium under transverse load. The famous solution, obtained from the energy balance principle, is given by

$$\sigma = \sqrt{\frac{2E\gamma}{\pi a}} \tag{1}$$

where σ is the far field stress causing the crack to open and grow unstably under plane stress conditions with "a" being the half crack length and ϒ being the classical surface energy due to the breakage of bonds in the generation of new crack surface.

In considering applications to glass it was found that the concept and form of (1) is correct and the prediction of the failure load level was reasonable using the classical surface energy ϒ for glass. However for other materials, especially ductile metals, their values for ϒ can greatly underestimate the actual energy required to extend the crack.

Much later in the 1950's Irwin generalized the form of (1) by introducing the macroscopic energy release rate, G, as an independent property. Thus for the same central crack problem

$$\sigma = \sqrt{\frac{EG}{\pi a}} \qquad (2)$$

where

$$G = \frac{\partial U}{\partial A} \qquad (3)$$

with U being the total potential and A the crack area.

Going even further, Irwin greatly expanded the utility and applicability of the method by introducing the stress intensity factor with

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) \qquad (4)$$

where this is the form of the stress field near the linear elastic square root singularity. K_I in the above mentioned central crack problem is given by

$$K_I = \sqrt{\pi a}\sigma \qquad (5)$$

In more general problems then

$$K_I = \alpha\sqrt{\pi a}\sigma \qquad (6)$$

for Mode I crack opening conditions. Similar forms follow for the two shear modes, II and III, and for plane strain. The values of α can be found for any problem of interest with characteristic dimension ”a”.

The criterion for crack instability is then given by

$$K_I^2 = EG \tag{7a}$$

or simply

$$K_I = K_{Ic} \tag{7b}$$

where K_{Ic} can be viewed as a property of the material, the fracture toughness.

The treatment just outlined follows from linear elastic analysis. Griffith provided the conceptual breakthrough and Irwin the follow through with an imaginatively organized, general methodology. This approach depends upon small scale yielding/damage as existing only in the vicinity of the crack tip and is referred to as linear elastic fracture mechanics.

The method recognizes that linear elastic singularities do not really exist, but still creatively uses them as the normalizing agent common to all cases of its type. The entire formalism is elegant, self contained, and it represented a huge step forward in understanding the failure of materials in the presence of stress magnifying flaws, defects and cutouts.

In effect, the above treatment represents the first part of the development of the field. The second part represents the approach to be taken when small scale yielding does not apply. That is, when the material yielding is over a region of about the same size as the crack or even over a larger region. This situation is especially important with ductile metals, and it will be summarized next. General treatments of fracture mechanics can be found in many sources such as Kanninen and Popelar [1], Broberg [2], Suo [3], and Anderson [4].

Continuing now with the case of large scale yielding, Rice [5] recognized that a wholly different and new approach would be required. He showed that the full plasticity problem could be simulated by the more direct nonlinear elasticity problem in the cases of proportional loading. Then with an innovative use of a path independent line integral, the J integral, he formulated the fracture problem in a completely different and more tractable manner. In the case of small scale yielding and for linear elasticity the J integral method gives the value of the contour integral as

$$J = \frac{K_I^2}{E} \tag{8}$$

This development then opened the door to using this method in the much more complicated nonlinear case (and linear as well) usually using power law forms to represent the nonlinear constitutive behavior. This theoretical synthesis by Rice of all the elements of fracture mechanics completely consolidated the field and further enlarged its utility, as shown by subsequent activity.

Expanding on this class of nonlinear problems, Hutchinson [6] and others developed a powerful approach and methodology for proceeding in general. Also, it should be observed that Rivlin and Thomas formulated fracture mechanics as explicitly applicable to highly nonlinear elastomers.

Much contemporary work has centered around models of the nonlinear conditions and the three dimensional effects in the crack tip regions, especially at interfaces involved in the debonding of dissimilar materials. All these developments and contributions aggregate to a complete and widely used methodology for treating crack instability failure.

The Two Failure Theories

Fracture mechanics thus provides a highly useful method for approaching and solving many failure problems. Alternatively many sources as well as this website show that failure criteria also provides a viable and comprehensive method for solving many failure problems. How should one approach the question of which formulation to use with a particular problem? The key to answering this question lies with the concept of homogeneity, which was extensively discussed in Section I.

At some sufficiently large scale, most of the standard materials classes are taken to be homogeneous. Usually this scale is the same order as the dominant scale of the applications of interest. At much smaller scales a new and vivid landscape of flaws, defects and irregularities are to be seen. But at the macroscopic scale of application there exists a complete suite of homogeneous material properties, the intrinsic mechanical and thermal properties, that control behavior at this most common scale.

It is at the common scale of homogeneity of the material that most failure problems are most usefully posed. Of course the macroscopic strength is profoundly affected by the subscale state of flaws, but not just by a single, idealized one of them. There is a whole distribution of them contributing to and causing failure at the macroscopic scale of the homogeneous material.

However, there is another independent scale of relevance and control that must also be considered. This scale emerges only in the particular problem of the intended application. This scale is that which implicitly governs the gradients of the applied stress state. If the gradients of the stress state (relative to the scale of the homogeneity of the material) are great, then fracture mechanics may be called for. If these stress gradients are shallow to moderate, then failure criteria are in order.

In the above context, failure criteria are defined and derived for the failure of homogeneous materials under homogeneous stress states. In effect failure criteria represents the completion of the constitutive specification for the homogeneous material. Of course the failure criteria and the other parts of the constitutive equations are then used in applications involving stress gradients so long as these gradients are not extreme. This is the simplest, most straightforward statement of the basis for the failure criteria methodology.

Is there an arbitrarily sharp division of scale between these two conditions of applicability – not likely. But it is usually fairly obvious which case is present in a given problem. Stress conditions around sharp cracks and corners belong to the first group. Most other problems without extreme geometric curvature features fall into the second group. Thus it is both the scale of the homogeneity of the material and the scale (gradient) of the possible inhomogeneity of the stress state, extreme versus moderate, that helps decide which theory to use.

Possibly all this makes the decision process seem more complicated than it should be or actually is. Good judgment usually is enough to render the decision. Two examples will now be given, one where fracture mechanics is obviously the correct approach and one where the use of failure criteria is clearly called for. Interest here is only with examples representing important, practical applications, not obscure problems of no relevance or of misleading status.

Fracture Mechanics Example

The problem illustrating the application of fracture mechanics is that of the critical size of an edge crack in a load carrying structural member. Typically cracks form and grow near the edges and surfaces that are formed in processing and fabrication stages. The explicit problem is that of an edge crack as shown in Fig. 1.

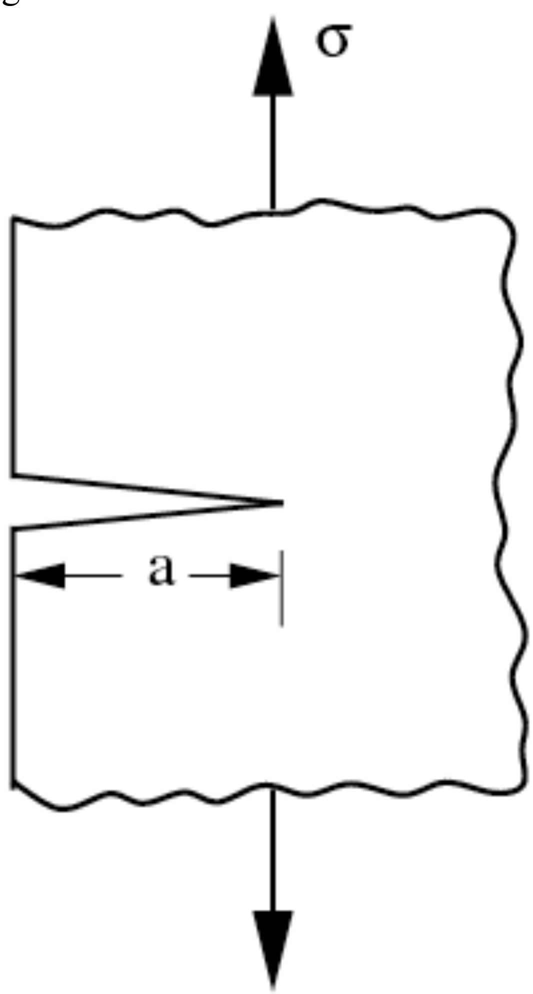

Fig. 1 Edge crack

Often cracks such as these grow under low level fatigue conditions. The question is at what crack size "a" does the crack under constant load become unstable and cause failure. The governing fracture mechanics form is (6) and for this problem of the edge crack it is given by the stress intensity factor

$$K_I = 1.12\sqrt{\pi}\sigma\sqrt{a} \tag{9}$$

To pose a specific problem, take the working stress level as

$$\sigma = 200\ MPa$$

For aluminum the critical stress intensity factor is about

$$K_{Ic} = 25\ MPa\sqrt{m}$$

Combining these last three equations gives the critical crack size "a" as

$$a = 3.97\ mm$$

This result is the critical crack size as found from linear elastic fracture mechanics. For significant plasticity behavior, as would be expected in this problem, the corresponding critical crack size would be less than this value, thus

$$a < 4\ mm$$

So it is found that the critical crack size for this problem would certainly be less than 1 cm, probably much less.

Although the explicit fracture mechanics formulas have been used here to estimate the critical crack size for a particular problem, that is not how it would normally be done in practice. Usually there are industry standards for the maximum allowable crack size in particular classes of problems and these have been established from extensive data bases. Nevertheless, this latter procedure still represents an explicit and critical use of fracture mechanics. In nearly all safety related applications there are very comprehensive programs and protocols for detecting dangerous cracks and flaws and damage in order to prevent sudden fracture mechanics types failures.

Failure Criterion Example

The problem illustrating the necessary use of failure criteria is that for the single most basic problem in composite materials technology. This problem is that of the single very stiff and strong spherical inclusion in an elastic medium under far field stress conditions, the dilute suspension case. This is the fundamental problem for the effective stiffness problem in elasticity as well as the effective viscosity for dilute fluid suspensions. The strength problem in the elasticity context is the complement of the effective stiffness problem.

To carry out this strength analysis, the three dimensional elasticity solution is needed for the infinitely stiff spherical inclusion in the infinite elastic medium under far field uniaxial tensile stress. Fig. 2 shows the problem of interest.

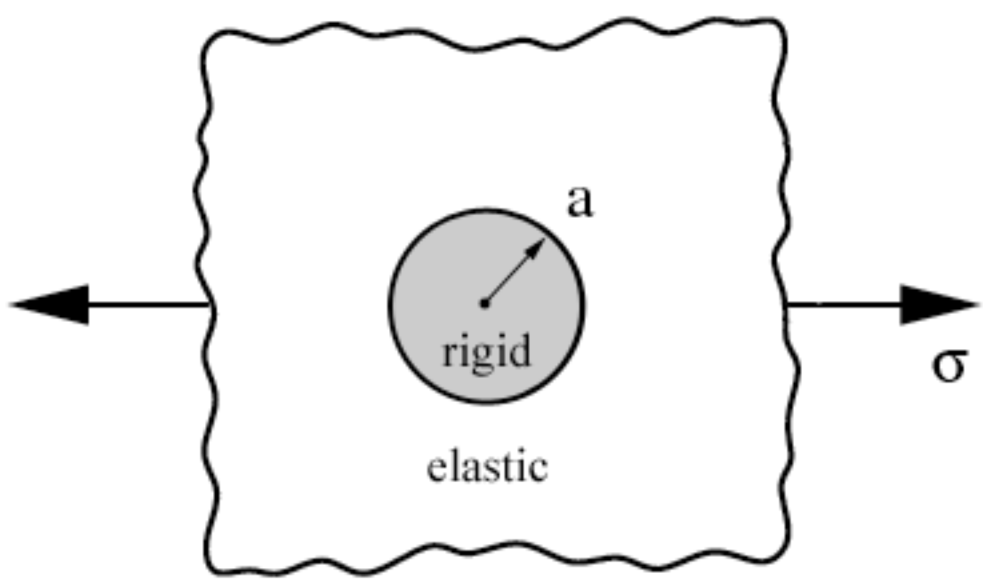

Fig. 2 Rigid spherical inclusion in an infinite elastic medium

Finding the exact elasticity solution is not a trivial exercise. The stresses in the elastic medium are found by solving the governing coupled partial differential equations following a similar method given by Christensen [7] for the effective shear stiffness of the same constituents.

Perfectly continuous interface conditions are assumed. In general the stresses are a maximum at the interface with the spherical inclusion of radius "a". In spherical coordinates with the θ=0 axis being in the direction of the far field uniaxial stress, σ, then at r=a the stresses are found to be

$$\sigma_r = \frac{3(1-\nu)}{(4-5\nu)}\left[\frac{3}{(1+\nu)} - \frac{5}{2}\sin^2\theta\right]\sigma$$

$$\sigma_\theta = \sigma_\phi = \frac{3\nu}{(4-5\nu)}\left[\frac{3}{(1+\nu)} - \frac{5}{2}\sin^2\theta\right]\sigma \tag{10}$$

$$\sigma_{r\theta} = -\frac{15(1-\nu)\sigma}{2(4-5\nu)}\sin\theta\cos\theta$$

The other two stress components vanish identically because of symmetry.

Often in 3-dimensional elasticity solutions the results take especially simple forms when Poisson's ratio has the value ν=1/5. In this case (10) becomes

$$\sigma_r = 2\sigma\cos^2\theta$$

$$\sigma_\theta = \sigma_\phi = \frac{\sigma}{2}\cos^2\theta \tag{11}$$

$$\sigma_{r\theta} = -2\sigma\sin\theta\cos\theta$$

The value ν=1/5 represents that for some glass and ceramic materials.

In composite materials, the very stiff and strong inclusions are usually used within a polymeric matrix phase. The strength problem therefore will be taken for a quite glassy polymeric matrix material such as a standard epoxy resin with typical properties of

$$\nu = \frac{2}{5} = 0.4$$

$$\frac{T}{C} = \frac{1}{2}$$

These properties and the stresses (10) at r=a and at θ=0 will be used in the general, isotropic material failure criterion, Eq. (1), Section VII, repeated here as

$$\left(1-\frac{T}{C}\right)\left(\hat{\sigma}_1+\hat{\sigma}_2+\hat{\sigma}_3\right)+ \frac{1}{2}\left[\left(\hat{\sigma}_1-\hat{\sigma}_2\right)^2+\left(\hat{\sigma}_2-\hat{\sigma}_3\right)^2+\left(\hat{\sigma}_3-\hat{\sigma}_1\right)^2\right] \le \frac{T}{C} \tag{12}$$

The principal stresses in (12) are normalized by the uniaxial compressive strength C. This procedure then gives the strength result for the problem of Fig. 2 as

$$\sigma = 0.428\,T$$

Thus the local failure occurs, and in this brittle circumstance likely leads to overall failure, when the applied far field uniaxial tensile stress is between 1/3 and 1/2 the value of the tensile strength of the polymeric matrix material. This is the effect of the stress concentration caused by the rigid inclusion. It can only be obtained quantitatively through the failure criterion since more than one component of stress is operative

In general, very stiff inclusions enhance the effective stiffness of the carrier matrix material. However, contrary to popular belief, in some cases and in this particular example the strength properties are degraded by the presence of the "reinforcing" inclusion(s). The problem becomes a trade-off between stiffness and strength.

Assessment

These two examples are typical of a great many realistic situations. There is a large array of very important problems that are covered by failure criteria, but there is an equally large and important collection covered by fracture mechanics. Neither approach can be said to be more important than the other. They both are vitally important, and the two fields constitute complementary approaches in solving and codifying the critical failure problems involved in materials applications.

So there are two independent methodologies for dealing with failure, fracture mechanics and failure criteria. The basic properties for both fields are needed in order to completely characterize the performance capability for any particular material in any particular application. It is interesting that there are parallel features shared by both of them. Fracture mechanics includes both brittle and ductile fracture behaviors, while failure criteria accommodates both ductile flow and brittle (and ductile) fracture for different materials types in different regions of stress space. To this extent there is an overlap between the two approaches. However, the differences are far greater than this superficial similarity.

Failure criteria, as developed here, is the rigorous theory of failure behavior for homogeneous materials under quasi-homogeneous (not extremely inhomogeneous) stress states. Fracture mechanics is an equally rigorous theory of failure behavior for the failure of structures (sometimes very simple structures) that always include a region of an extremely inhomogeneous stress state, the stress intensity zone, surrounding a crack or crack-like boundary condition. The term "structures" is used here rather than "materials" because boundary value problems are involved in the determination of the stress intensity factors. Both theories are often used beyond the narrow range of their derivations and this becomes a matter of experience and judgment in applications.

Will there ever be a complete, general theory of failure that subsumes both of these approaches? It may be possible but it is very unlikely. Attempting such a unified development would be an exceedingly difficult undertaking. There is great utility as well as considerable beauty in these two carefully constructed, carefully circumscribed, simpler theories: fracture mechanics and failure criteria.

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