XX. FIRST PLY FAILURE AS DETERMINED BY THE MOST RIGOROUS (YET SIMPLEST) FIBER COMPOSITES FAILURE CRITERION AND FAILURE THEORY

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Introduction

By far the most prominent recent activity on the failure of fiber composite materials has been the World Wide Failure Exercise, WWFE. Many publications have been involved in the exercise over the course of many years. All can be accessed through the WWFE-II, Hinton and Kaddour [1]. The high profile WWFE program provides a suitable starting position for the investigation to be given here.

The WWFE was originally chartered to straighten out the existing state of chaos with composites failure and it was expected to converge to the best general model of failure. It did not do so. There could be many contributing reasons for this but one of them was a complete reliance on test data of unproven reliability. This then lead to a rigid adherence on quantifying how close each and every failure model of the many considered could replicate the data. Metrics were devised to establish the relative merit of each model on that basis. The inevitable consequence of this was to favor and promote models with many parameters and extremely broad flexibility. The model judged to be the best was said to have an incredible 50 parameters. Others had even more. The so judged top 4 models (all of them parameters intensive) were actually even declared to be nearly ready for general applications.

Such models are totally empirical and add little or nothing to the understanding of composites failure. There were no special insights or revealing developments in that approach. There probably was much of good intention in the initial effort but it ended up being obscured by the misguided approach to evaluation. See Christensen [2] for a very detailed critical documentation of the entire WWFE program.

Despite its shortcomings the WWFE does offer some lessons on where to go, or at least where not to go. Most importantly the WWFE raises the following fundamental question. Is it possible to develop and derive a rational and general failure criterion composed only of basic and standard strength properties of the material and completely devoid of adjustable parameters and/or unjustified assumptions? Of course this refers to aligned fiber composite materials, but the same bottom line question could be asked of isotropic materials as well. Essentially this question asks if it is possible to pursue materials failure as a scientific investigation as opposed to a parameters exercise. The discourse here will begin with the failure of isotropic materials as providing the absolutely necessary background for the prime interest in fiber composite materials.

Finally, it should be recognized that any new fiber composites failure theory remains un-validated without experimental evaluation.` This paper lays out the full failure theory and then finishes with a demonstration and application to first ply failure. The experimental evaluation will come in the second and following paper which will use this new failure theory to predict total failure of the laminate. It is only in the case of total failure where there is reliable and significant experimental data.

Isotropic Materials Failure

It could hardly be imagined that one could correctly treat the failure of fiber reinforced composite materials without first having a thorough understanding of the failure of isotropic materials. Unfortunately the failure of isotropic materials has had a long and difficult and frustrating history. Over the span of history, isotropic materials failure has gone through the same phases of frustration as has occurred with composites in recent years. Only the yielding of ductile metals ever was or now is well understood. For more general isotropic materials, over the long term, failure has been a complete mystery with sporadic periods and bursts of great activity but no successful results. Only very recently has the subject yielded to a rational treatment, ultimately giving an astonishingly simple form for its failure theory.

The theory of failure for isotropic materials was recently developed by Christensen and presented in the book on the subject, [3]. The theory was fully verified and thoroughly treated in Refs. [3] and [4]. The main form of the isotropic materials failure theory that is of relevance to fiber composites is the polynomial invariants method of deriving the failure criteria. The end result forms will be stated here as necessary background for the fiber composites case.

The polynomial invariants failure criterion for isotropic materials is given by

$$\left(1-\frac{T}{C}\right)\hat{\sigma}_{ii}+\frac{3}{2}\hat{s}_{ij}\hat{s}_{ij}\leq\frac{T}{C} \tag{1}$$

where T and C are the uniaxial tensile and compressive strengths. The first term in (1) is the first invariant of the stress tensor and the second term is the second invariant. Symbol s_{ij} is the deviatoric stress tensor and the stresses are nondimensionalized by the compressive strength C,

$$\hat{\sigma}_{ij}=\frac{\sigma_{ij}}{C} \tag{2}$$

Remarkably the failure theory is fully calibrated by only two strength properties T and C. There are no empirical parameters.

The failure criterion (1) when written in component form is from Eq. (4.16) of Ref. [3] as

$$\begin{aligned}\left(\frac{1}{T}-\frac{1}{C}\right)&(\sigma_{11}+\sigma_{22}+\sigma_{33})\\ &+\frac{1}{2TC}[(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2\\ &+6(\sigma_{12}^2+\sigma_{23}^2+\sigma_{31}^2)]\leq 1\end{aligned} \tag{3}$$

where stress is now stated in dimensional form.

It is not possible to specify the shear stress at failure, S, independently of T and C. It is uniquely determined by the failure criterion (3) by taking all stresses as zero except $\sigma_{12}=S$, then yielding

$$S^2=\frac{TC}{3} \tag{4}$$

The fact that S is not independent of T and C will have a direct and fundamentally important counterpart in the coming theory of failure for fiber composites.

The failure criterion (1), (3) is the rigorous form to be used for the failure of the isotropic polymeric matrix phase in fiber composites. This is a further deep and significant tie between the theory of failure for isotropic materials and that of fiber composite materials.

Finally with regard to isotropic materials failure, it is of importance to recognize that the failure theory is fully calibrated by only two strength properties, the same as the number of independent elastic properties for isotropy. The fact that the number of independent failure properties is the same as the number of independent elastic properties is not just a coincidence. It is inherent in and a direct consequence of the method of derivation of the failure criteria by the polynomial invariants method, see Ref. [3]. This basic relationship will have important consequences for the case of fiber composite materials, to be given next.

With the successful treatment of the failure of general, isotropic materials, there finally is a realistic argument for and solid guidance for proceeding to treat the failure of anisotropic fiber composite materials.

Unidirectional Fiber Composites Failure

The unidirectional fiber composites failure criteria are derived by the same method, the "failure invariants theory", FAIT, as was used with isotropic materials in the preceding section. The steps of the derivation will be given next.

The failure criteria are for highly anisotropic fiber composites. The restriction to the condition of high anisotropy applies to both stiffness and strength. This condition is appropriate for carbon fiber polymeric matrix composites but it may not apply to glass fiber composites. The explicit polynomial invariants method for the highly anisotropic conditions for transversely isotropic symmetry requires decomposition of the failure criteria into two parts, as derived in Refs. [3] and [5]. These are the fiber controlled criterion and the matrix controlled criterion as given by

Fiber Controlled Failure

$$-C_{11} \leq \sigma_{11} \leq T_{11} \tag{5}$$

where axis 1 is in the fiber direction and T_{11} and C_{11} are the fiber direction tensile and compressive strengths. The matrix controlled failure criterion is given by

Matrix Controlled Failure

$$\left(\frac{1}{T_{22}} - \frac{1}{C_{22}}\right)(\sigma_{22} + \sigma_{33}) + \frac{1}{T_{22}C_{22}}(\sigma_{22} + \sigma_{33})^2 + \frac{1}{S_{23}^2}(\sigma_{23}^2 - \sigma_{22}\sigma_{33}) + \frac{1}{S_{12}^2}(\sigma_{12}^2 + \sigma_{31}^2) \leq 1 \tag{6a}$$

where

$$S_{23}^2 = \left(\frac{1 + \frac{T_{22}}{C_{22}}}{3 + 5\frac{T_{22}}{C_{22}}}\right) T_{22}C_{22} \tag{6b}$$

In principal stress space σ_2, σ_3 of the 2-3 plane the failure criteria (6a) and (6b) combine to give the especially simple single form for the matrix controlled failure as

$$\left(\frac{1}{T_{22}} - \frac{1}{C_{22}}\right)(\sigma_2 + \sigma_3) + \frac{1}{T_{22}C_{22}}\left[\sigma_2^2 + \sigma_3^2 - \left(\frac{1 + 3\frac{T_{22}}{C_{22}}}{1 + \frac{T_{22}}{C_{22}}}\right)\sigma_2\sigma_3\right] + \frac{(\sigma_{12}^2 + \sigma_{31}^2)}{S_{12}^2} \leq 1 \tag{6c}$$

In (6a), (6b), and (6c) T_{22} and C_{22} are the transverse tensile and compressive strengths and S_{12} is the axial shear strength.

As proven in Ref. [5] the transverse shear strength S_{23} is not an independent strength property but is determined by T_{22} and C_{22}. This is in complete consistency with the derivation of the isotropic materials failure criterion by FAIT, giving the isotropic result for the shear strength as (4).

The range of the T_{22} and C_{22} strength properties in (6a)-(6c) is given by

$$0 \leq \frac{T_{22}}{C_{22}} \leq 1 \tag{7}$$

Using (7) in (6b) then requires

$$\frac{1}{4} \leq \frac{S_{23}^2}{T_{22}C_{22}} \leq \frac{1}{3} \tag{8}$$

or

$$0.5 \leq \frac{S_{23}}{\sqrt{T_{22}C_{22}}} \leq 0.577 \tag{9}$$

It would be virtually impossible to always determine S_{23} experimentally to the accuracy required by (9). S_{23} is a prediction from the theory and extremely difficult to determine experimentally as an independent entity.

The most common value for the strength ratio $\frac{T_{22}}{C_{22}}$ for carbon fiber polymeric matrix composites is $\frac{T_{22}}{C_{22}} = \frac{1}{3}$, giving from (6b)

$$At\ \frac{T_{22}}{C_{22}} = \frac{1}{3}, \qquad S_{23}^2 = \frac{2}{7}T_{22}C_{22} \tag{10}$$

The two limits in (8) and the common value of S_{23}^2 in (10) give the sequence

$$\frac{2}{6}, \frac{2}{7}, \frac{2}{8}$$

showing the regularity of behavior.

The narrow window on S_{23} would be unappealing and irrelevant from a many parameters approach point of view. But from a physical point of view it is advantageous. The theory itself is telling us the permissible and the impermissible ranges for properties and behaviors. It is a highly significant advantage of the polynomial invariants theory.

The fiber composites failure criteria in (5) and (6) are very easy to use. The fiber controlled criterion (5) is identical with the common, intuitive form. For some this may seem too simple to be realistic, but that would be incorrect reasoning. It is the rigorous result that comes directly from the polynomial invariants theory method in the case of highly anisotropic fiber composites. The two failure criteria are the most rigorous forms in existence. They follow from a rational derivation rather than merely being postulated, as are most failure criteria.

The failure criteria (5) and (6) are also the simplest ones in existence since they only require 5 strength properties to calibrate them. Most failure criteria require a large number of parameters for calibration. This fiber composites failure theory is calibrated by only 5 strength properties, the same as the number of independent elastic properties for transverse isotropy. This is in complete harmony with the corresponding behavior of isotropic materials failure found by FAIT and given in the preceding section.

It is physically helpful to examine the case where a transverse pressure P is applied and increased until failure occurs. This then is a matrix controlled failure and it follows directly from (6a) and (6b) or equivalently (6c). Let

$$\sigma_{22} = \sigma_{33} = -P \tag{11}$$

The solution for the transverse pressure at failure is found to be

$$\frac{P}{C_{22}} = \left(1 + \frac{T_{22}}{C_{22}}\right) + \sqrt{\frac{1 + 2\left(\frac{T_{22}}{C_{22}}\right) - \left(\frac{T_{22}}{C_{22}}\right)^2}{1 - \frac{T_{22}}{C_{22}}}} \tag{12}$$

At the common value of $\frac{T_{22}}{C_{22}}$ for carbon fiber polymeric matrix composites

$$At\ \frac{T_{22}}{C_{22}} = \frac{1}{3}, \qquad \frac{P}{C_{22}} = \frac{1}{3}\left(4+\sqrt{22}\right) = 2.90 \tag{13}$$

The result (12) and the particular case (13) are revealing. Pressure P at failure must be large compared with C_{22} but likely not an order of magnitude larger. The result here that $P \cong 3C_{22}$ is perfectly reasonable. This would be an excellent test for any composites failure theory. Can it reasonably predict this transverse pressure failure without any "fudge factors" (sliding parameters)?

The transverse pressure failure problem and solution just given can be enlarged to include the axial shear stress, σ_{12}. This new problem is to determine the axial shear stress at failure under transverse pressure P. The solution from, (6c) is given by

$$\left(\frac{\sigma_{12}}{S_{12}}\right)^2 = 1 + 2\left(1-\frac{T_{22}}{C_{22}}\right)\left(\frac{P}{T_{22}}\right) - \left(\frac{T_{22}}{C_{22}}\right)\left(\frac{1-\frac{T_{22}}{C_{22}}}{1+\frac{T_{22}}{C_{22}}}\right)\left(\frac{P}{T_{22}}\right)^2 \tag{14}$$

As a typical example, again take $\frac{T_{22}}{C_{22}}$ as

$$At\ \frac{T_{22}}{C_{22}} = \frac{1}{3}, \qquad \left(\frac{\sigma_{12}}{S_{12}}\right)^2 = 1 + \frac{4}{3}\left(\frac{P}{T_{22}}\right) - \frac{1}{6}\left(\frac{P}{T_{22}}\right)^2 \tag{15}$$

For the reasonable pressure level of

$$\frac{P}{C_{22}} = \frac{1}{2}, \qquad \frac{P}{T_{22}} = \frac{3}{2}$$

then

$$\left(\frac{\sigma_{12}}{S_{12}}\right)^2 = \frac{21}{8} \tag{16}$$

$$\left(\frac{\sigma_{12}}{S_{12}}\right) = 1.620 \tag{17}$$

Thus it is seen that at a transverse pressure of $P = \frac{C_{22}}{2}$ the axial shear strength is increased by 62%. It is well know that superimposed pressure has a profound effect upon the strength in most stress states and this failure theory verifies and quantifies that general effect for fiber composites

The axial shear stress squared at failure in (14) increases with increasing transverse pressure, reaching a maximum at the pressure

$$\frac{P}{C_{22}} = 1 + \frac{T_{22}}{C_{22}}$$

and thereafter decreasing until reaching failure under the transverse pressure by itself.

There isn't much doubt that a failure model with an abundance of parameters can have some success in fitting data sets of uncertain validity. But a blind adherence to that approach is not likely to endure. The only possibility for permanence is the physically based failure theory built up from steps of logic in its formulation and with no unspecified parameters. It will be the one most likely to model most physical features of realistic behavior, such as those shown here.

First Ply Failure in a Quasi-Isotropic Laminate

First ply failure will be determined for a quasi-isotropic laminate. The failure criteria (5) and (6) will be used to determine whether the initial failure is due to matrix controlled failure or due to fiber controlled failure and in which lamina the first failure actually occurs for a given stress state. This is the age old question but the answer is uniquely different for each different failure criterion under consideration and evaluation. With regard to the matrix controlled criterion (6) the failure could be due to either explicit matrix failure or interface failure. Both cases are covered by the failure criterion (6).

The quasi-isotropic laminate form has the 4 plies directions as shown and designated in Fig. 1.

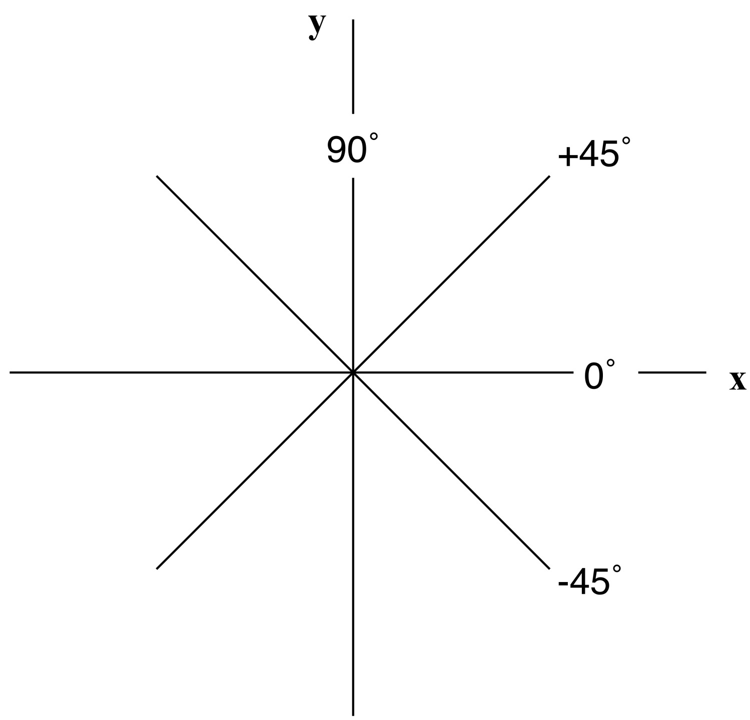

Fig. 1 Laminate conventions

At the lamina level the 1,2,3 axes are used with 1 in the fiber direction.

The lamina level criteria from (5) and (6) take the forms

Fiber Controlled Failure

$$\sigma_{11} = T_{11} \quad and \quad \sigma_{11} = -C_{11} \tag{18}$$

Matrix Controlled Failure

$$\left(\frac{1}{T_{22}} - \frac{1}{C_{22}}\right)\sigma_{22} + \frac{\sigma_{22}^2}{T_{22}C_{22}} + \frac{\sigma_{12}^2}{S_{12}^2} = 1 \tag{19}$$

The lamina level failures involve the stresses as given by

0˚ Lamina

$$\sigma_{11} = \frac{E_{11}}{E(1-\nu_{12}\nu_{21})}\left[(1-\nu\nu_{21})\sigma_x + (\nu_{21}-\nu)\sigma_y\right] \tag{20}$$

$$\sigma_{22} = \frac{E_{22}}{E(1-\nu_{12}\nu_{21})}\left[(\nu_{12}-\nu)\sigma_x + (1-\nu\nu_{12})\sigma_y\right]$$

$$\sigma_{12} = 0$$

90˚ Lamina

$$\sigma_{11} = \frac{E_{11}}{E(1-\nu_{12}\nu_{21})}\left[(\nu_{21}-\nu)\sigma_x + (1-\nu\nu_{21})\sigma_y\right]$$

$$\sigma_{22} = \frac{E_{22}}{E(1-\nu_{12}\nu_{21})}\left[(1-\nu\nu_{12})\sigma_x + (\nu_{12}-\nu)\sigma_y\right] \tag{21}$$

$$\sigma_{12} = 0$$

+45˚ Lamina

$$\sigma_{11} = \frac{(1-\nu)(1+\nu_{21})E_{11}}{2(1-\nu_{12}\nu_{21})E}\left(\sigma_x + \sigma_y\right)$$

$$\sigma_{22} = \frac{(1-\nu)(1+\nu_{12})E_{22}}{2(1-\nu_{12}\nu_{21})E}\left(\sigma_x + \sigma_y\right) \tag{22}$$

$$\sigma_{12} = \frac{(1+\nu)\mu_{12}}{E}\left(\sigma_x - \sigma_y\right)$$

-45˚ Lamina

$$\sigma_{11} = \frac{(1-\nu)(1+\nu_{21})E_{11}}{2(1-\nu_{12}\nu_{21})E}\left(\sigma_x + \sigma_y\right)$$

$$\sigma_{22} = \frac{(1-\nu)(1+\nu_{12})E_{22}}{2(1-\nu_{12}\nu_{21})E}\left(\sigma_x + \sigma_y\right) \tag{23}$$

$$\sigma_{12} = \frac{(1+\nu)\mu_{12}}{E}\left(-\sigma_x + \sigma_y\right)$$

The stress notations and conventions are the usual ones.

For the quasi-isotropic laminate, the modulus E and Poisson's ratio ν are given by

$$\nu = \frac{Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}}{3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}} \tag{24}$$

and

$$E = \frac{1}{8}(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66})(1 - \nu^2) \tag{25}$$

where

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}} \tag{26}$$

$$Q_{12} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{66} = \mu_{12}$$

and

$$\nu_{21} = \frac{E_{22}}{E_{11}}\nu_{12}$$

The lamina level elastic and failure properties for a realistic example are the typical values for a carbon/epoxy composite as

$$
\begin{aligned}
E_{11} &= 150 \text{ GPa} \\
E_{22} &= 9 \text{ GPa} \\
\mu_{12} &= 6 \text{ GPa} \\
\nu_{12} &= \frac{1}{3} \\
\nu_{23} &= \frac{1}{2}
\end{aligned} \tag{27}
$$

and

$$
\begin{aligned}
T_{11} &= 2000 \text{ MPa} \\
C_{11} &= 1500 \text{ MPa} \\
T_{22} &= 50 \text{ MPa} \\
C_{22} &= 150 \text{ MPa} \\
S_{12} &= 80 \text{ MPa}
\end{aligned} \tag{28}
$$

The elastic property ν_{23} is not needed in the example.

In the laminate stress space of σ_x and σ_y the first ply failure due only to the fiber controlled failure modes are shown in Fig. 2

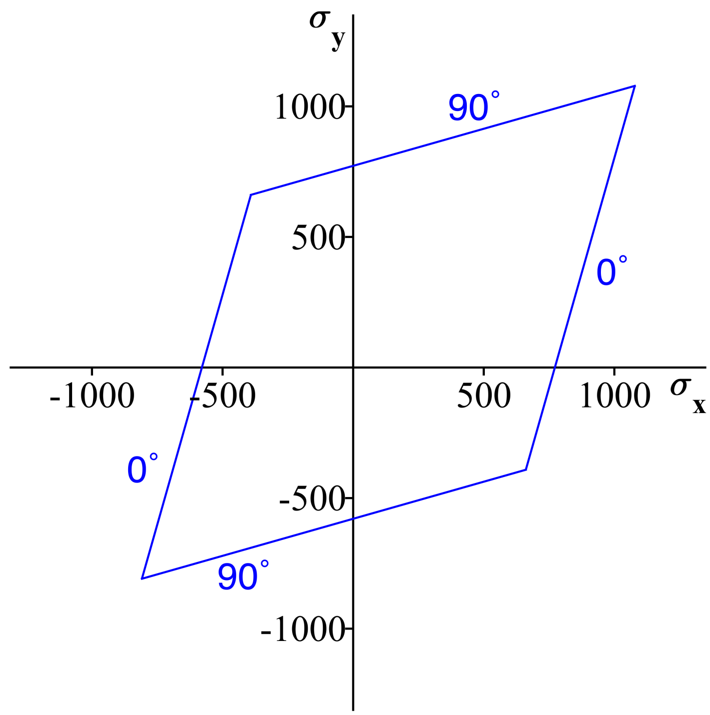

Fig. 2 Fiber controlled first ply failure

The matrix controlled failure modes by themselves are as shown in Fig. 3

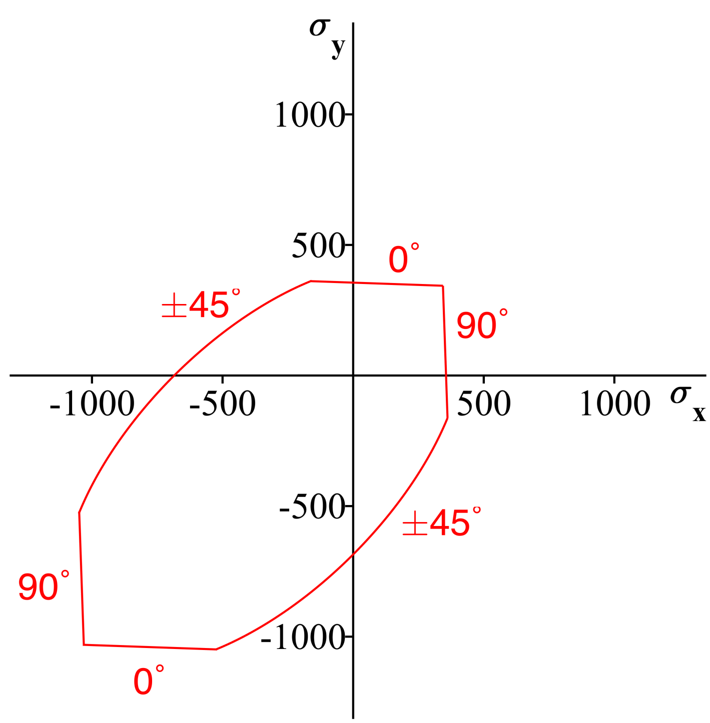

Fig. 3 Matrix controlled first ply failure

The most limiting combination of the fiber controlled and the matrix controlled failure modes for the laminate are as shown in Fig. 4

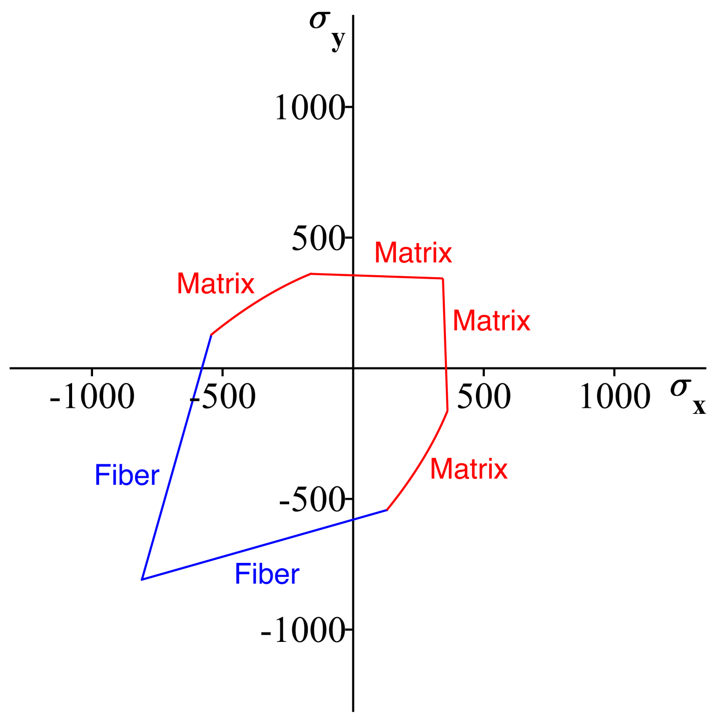

Fig. 4 First ply failures for the quasi-isotropic laminate

The Fig. 4 envelope is the complete first ply failure envelope for the entire laminate. The majority of the failure restrictions are due to the matrix failure but the region in the 3rd quadrant is limited by the fiber compressive failure mode.

This particular case shows the ease with which the problem of 1st ply failure can be executed for fiber composite laminates. First ply failure is often the most limiting condition for design considerations. The important thing to remember is that the results are no better than the lamina level, unidirectional fiber composites failure theory from which they are generated. Such problems can become quite complex for the more general three-dimensional problems often involved with composites but they still are directly amenable to solution.

Also the general anisotropic problems at the laminate level become more involved than that given here for the quasi-isotropic case. Nevertheless the quasi-isotropic case is the most important problem of all. It must be solved first before the more challenging problems can be approached. Until there is agreement on the proper failure criteria for the unidirectional composites case and its transfer to the laminate level (first for the quasi-isotropic laminate) the state of high confusion with composites failure will persist and continue.

Conclusions

A rationally derived and evaluated failure criterion for unidirectional fiber composites has been given. It should be compared with the norms in the field such as the top rated theories in the World Wide Failure Exercise [1]. These theories of failure as given by Pinho [6], Carrere [7], Puck [8] and Cuntze [9] involve multitudes of adjustable parameers. If the parameters approach were taken for elasticity behavior rather than its classical theoretical foundation, the result would certainly be completely worthless. There is no reason to believe it would be any more successful for failure. The present rigorous failure theory presents a stark alternative to the parameters approach and it offers an opportunity for fiber composites to move ahead and provide a more substantial basis for general improvement and optimization.

There will be a following paper related to this one that will take up the much more difficult problem of total failure of the laminate as opposed to that of first ply failure. The total failure predictions from this failure theory will then be compared with well established experimental data. The two problems, first ply failure and total failure, will be compared and assessed, and integrated. Further new insights and conclusions will be given on the general problem of the failure of fiber composite materials. In particular the first ply fiber failure concept will be completely discredited but the first ply matrix controlled failure (damage) as given here will be found to be well based and useful.

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