III. FAILURE CRITERIA FOR ANISOTROPIC FIBER COMPOSITE MATERIALS

The anisotropic materials types to be considered here will be taken to have transversely isotropic symmetry. Perhaps the best known example is that of aligned fiber composite materials, but there are many other examples. A further condition will be taken such that the degree of anisotropy is large. This is in line with the interests here in high stiffness and high strength fiber composite materials as typified by carbon fiber, polymeric matrix systems. Such materials will be referred to as carbon-polymer systems.

It is necessary to deduce the proper scale for the corresponding idealization of homogeneity for this class of materials failure problems. There are three obvious choices. The so called micromechanics level takes the individual fibers and the separate matrix phase in between them as the size scale for homogeneity. The next level up is the aligned fiber, lamina level, which then is much larger than the size of the individual filament or fiber. Finally, at yet a still much larger scale, the homogenization could be taken at the laminate level, involving the stacking of various lamina in various directions. It is the intermediate scale, the lamina level that is seen as having the proper balance between small scale detail, but large enough scale to include all the possible failure mechanisms which could be operative. An example of the importance of the scale of the failure mode will be given later. Thus all idealizations to follow are taken at the aligned fiber, lamina scale of homogenization. This is the same scale as that at which the volume averaged elastic properties for fiber composites are normally rationalized.

The main purpose here is to develop the highly anisotropic failure criterion (for carbon-polymer systems) which is the companion piece to that of the isotropic case given in the previous section. To this end, take a polynomial expansion of the stress tensor through terms of second degree for transversely isotropic symmetry. Such an expansion will involve the following seven terms composed of the four basic invariants for this symmetry and the three quadratic combinations of the two linear invariants,

$$\sigma_{11}\,, \quad (\sigma_{22}+\sigma_{33})\,, \quad \sigma_{11}^2\,, \quad (\sigma_{22}+\sigma_{33})^2\,,$$
$$\sigma_{11}(\sigma_{22}+\sigma_{33})\,, \quad \left(\sigma_{23}^2-\sigma_{22}\sigma_{33}\right), \quad \left(\sigma_{12}^2+\sigma_{31}^2\right) \tag{1}$$

Axis 1 is the axis of symmetry (fiber direction) and axes 2 and 3 form the plane of two dimensional isotropy.

The condition of high anisotropy will be taken for both stiffness and strength, thus for the moduli

$$\frac{E_{11}}{E_{22}} >> 1 \tag{2}$$

where E_{11} is the usual modulus in the fiber direction and E_{22} is the transverse modulus. For strengths, take T_{11} and C_{11} as the uniaxial tensile and compressive strengths in the fiber direction and T_{22} and C_{22} as the corresponding strengths in the transverse direction. The highly anisotropic strengths are specified by

$$\frac{T_{11}}{T_{22}} >> 1 \tag{3}$$

and

$$\frac{C_{11}}{C_{22}} >> 1 \tag{4}$$

The lamina level strength properties that are conventionally measured by standard tests are the six following as

$$T_{11}\,,\quad C_{11}\,,\quad T_{22}\,,\quad C_{22}\,,\quad S_{12}\quad and\quad S_{23} \tag{5}$$

where the T's and C's are already defined and S_{23} is the transverse shear strength while $S_{12} = S_{31}$ is the longitudinal or axial shear strength. The conditions of high anisotropy also imply that S_{12} and S_{23} are of much smaller size than are T_{11} and C_{11}.

With this terminology and the highly anisotropic stiffness and strength conditions, the failure criteria can now be formulated. The failure will be found to naturally decompose into two separate modes.

First consider the conceptual limiting case of rigid fibers. Insofar as the matrix phase is concerned, the physical state would be that of plane strain or alternatively that of out of plane shear deformation. For these stress states the effective macroscopic stresses controlling failure in the matrix phase would be those of the stresses in the 2-3 plane and the out of plane shear stresses.

For the cases of interest here where the fibers are not rigid but still are very stiff, (2), the stress forms resulting in matrix failure are taken to be the same as those in the above limiting case. The physical rationale for this is as follows. The anisotropic moduli ratio varies as $0 \le E_{22}/E_{11} \le 1$. The "0" limit is the plane strain case and the "1" limit is that of isotropy. Just as a value of $E_{22}/E_{11} = 0.9$ would have the isotropic case as a close and reasonable representation, so too the conjugate value of $E_{22}/E_{11}= 0.1$ would have the plane strain form as a close and reasonable representation. The value $E_{22}/E_{11}= 0.1$ or even considerably smaller values are well descriptive of typical carbon-polymer systems.

Taking the terms in (1) with these 2-3 plane components of stress and the out of plane shear stress components then gives the polynomial expansion as being comprised of the second, fourth, sixth and seventh terms in (1). This gives the matrix controlled failure criterion as,

$$\alpha(\sigma_{22}+\sigma_{33})+\beta(\sigma_{22}+\sigma_{33})^2+ \\ \gamma(\sigma_{23}^2-\sigma_{22}\sigma_{33})+\delta(\sigma_{12}^2+\sigma_{31}^2)\leq 1 \tag{6}$$

Evaluating parameters α, β, γ, and δ in (6) to give failure calibrated by the strength properties in (5) then gives the resulting failure criterion as :

Matrix Controlled Failure

$$\left(\frac{1}{T_{22}}-\frac{1}{C_{22}}\right)(\sigma_{22}+\sigma_{33})+\frac{1}{T_{22}C_{22}}(\sigma_{22}+\sigma_{33})^2+ \\ \frac{1}{S_{23}^2}(\sigma_{23}^2-\sigma_{22}\sigma_{33})+\frac{1}{S_{12}^2}(\sigma_{12}^2+\sigma_{31}^2)\leq 1 \tag{7}$$

For (7) to always have real roots it is necessary that $S_{23}\geq\frac{1}{2}\sqrt{T_{22}C_{22}}$. As with isotropic materials it is commonly found that $T_{22} < C_{22}$ from tests. In Section IX micromechanics is used to express S_{23} in terms of T_{22} & C_{22}.

The form (7) controls one of the two modes of failure. The other possible mode of failure is that of fiber controlled failure. It could be tempting to assume that this means that fiber failure itself is the limiting failure mechanism. Such is not usually the case however. The problem is far more complex and subtle than that. Consider for example the compressive failure due to stress in the fiber direction, σ_{11}. The formation of kink bands is usually the failure mechanism. With the high fiber stiffness, relation (2), the compressive failure mechanism is kink band formation with almost no deformation in the fiber direction, but the kink mechanism causes high shear stress in the matrix phase. The kink band formation occurs at the lamina scale, not the smaller scale, nor the larger scale. The kink occurs suddenly as an instability which has the stress

σ_{11} as proportional to the axial shear modulus. Even though the basic mechanism is that of an instability involving matrix deformation, it is still legitimate to designate this as fiber controlled failure since the high fiber stiffness plays an essential role, and the large failure stress is certainly related to the large fiber modulus. The situation with axial tensile failure is similarly complex with fiber breaks and fiber misalignment playing crucial roles. The Rosen model includes some of these effects in a two dimensional idealization.

The second mode of failure necessarily involves the other terms in (1) that are not part of the matrix controlled failure, (7). These terms are then those of

$$\sigma_{11}\,, \quad \sigma_{11}^2\,, \quad \sigma_{11}(\sigma_{22}+\sigma_{33}) \tag{8}$$

The large degree of anisotropy in strengths, (3) and (4), dictates that the fiber controlled failure envelope have stress states with σ_{22} and σ_{33} being much smaller than σ_{11}. With this condition, then the last term in (8) is negligible (but not vanishing) compared with the first two terms. Writing the first two terms in failure criterion form and evaluating the two parameters in terms of the strength properties in (5) gives the resulting failure criterion as :

<u>Fiber Controlled Failure</u>

$$\left(\frac{1}{T_{11}}-\frac{1}{C_{11}}\right)\sigma_{11}+\frac{1}{T_{11}C_{11}}\sigma_{11}^2 \le 1 \tag{9a}$$

or simply

$$-C_{11} \le \sigma_{11} \le T_{11} \tag{9b}$$

In contrast to the matrix controlled failure situation, it is commonly found that $C_{11} < T_{11}$.

The decomposed failure criteria (7) and (9) are completely calibrated by the six standard strength properties in (5). The fact that the fifth term of the expansion in (1) involving $\sigma_{11}(\sigma_{22}+\sigma_{33})$ does not enter either of these failure criteria is not an assumption, but rather the result of the rigorous derivation.

The background on the development of failure criteria (7) and (9) is as follows. The general outline of the method given here is similar to that developed by Christensen [1]. The end result, however, is different. In the references given, special assumptions were made which reduced the failure criteria to four or five property forms. With interest in generality, no such assumptions are used here, leaving the properties count as six. The paired failure criteria (7) and (9) are derived and displayed here for the first time, as of the date of this internet entry.

It should be noted that for applications the fiber direction stress σ_{11} must be taken in the fiber direction in the deformed configuration, not the reference configuration. To do otherwise could cause σ_{11} to induce a very large longitudinal shear stress which would certainly cause matrix controlled failure.

The physical significance of the present anisotropic failure criteria, (7) and (9), is that they are the direct counterpart of the isotropic failure criteria given in the previous section. As seen from the previous section, failure in isotropic materials is much more highly developed than is the anisotropic case. Nevertheless, as shown here, significant progress has been made in the more difficult anisotropic case.

The fiber controlled failure form (9) has an interesting history. This form is commonly called the maximum stress criterion. It has always been considered to be a highly useful but totally empirical form for fiber composites. In the present derivation it is not empirical at all, it is a rational and rigorous result of the method, which is based upon the conditions of a high degree of anisotropy, with no subsidiary assumptions. In this connection it can also be observed that the matrix controlled failure criterion, (7), certainly is not a maximum stress form. The two criteria (7) and (9) seem to be very different when compared through the commonly used form (9b) but not so greatly different when compared through the equivalent but more formal (9a). The two coordinated failure criteria, (7) and (9), are the end result of this physical derivation.

The failure criteria (7) and (9) are thus fundamentally based upon the high degree of anisotropy conditions (2)-(4), which in turn are motivated by the properties of carbon-polymer systems. Although some other types of fiber composites may not satisfy the high anisotropy conditions, they likely would still favor the separation of failure modes as in (7) and (9). Most systems at high fiber concentration have failure modes strongly influenced by the fiber to matrix morphology. As an example, transverse cracking as a matrix controlled failure mode is common to virtually all fiber composites.

Two well known failure criteria for fiber composites are those of the Tsai-Wu form and the Hashin form. These two criteria will be stated here for comparison with (7) and (9).

The Hashin criterion [2] starts with the same seven terms in (1), and then decomposes into separate fiber and matrix failure modes. It also distinguishes essentially tensile states from compressive states, with separate criteria taken for each. This then involves many more terms than just those in (1) being used singly, since some are used twice. Finally, several assumptions are used to bring the parameter count down to a manageable level of the six properties in (5). The end result is the Hashin criterion given by :

Tensile Matrix Mode, $(\sigma_{22}+\sigma_{33}) > 0$

$$\frac{1}{T_{22}^2}\left(\sigma_{22}+\sigma_{33}\right)^2+\frac{1}{S_{23}^2}\left(\sigma_{23}^2-\sigma_{22}\sigma_{33}\right)+\frac{1}{S_{12}^2}\left(\sigma_{12}^2+\sigma_{31}^2\right)\leq 1$$

Compressive Matrix Mode, $(\sigma_{22}+\sigma_{33}) < 0$

$$\frac{1}{C_{22}}\left[\left(\frac{C_{22}}{2S_{23}}\right)^2-1\right](\sigma_{22}+\sigma_{33})+\frac{1}{4S_{23}^2}\left(\sigma_{22}+\sigma_{33}\right)^2+$$

$$\frac{1}{S_{23}^2}\left(\sigma_{23}^2-\sigma_{22}\sigma_{33}\right)+\frac{1}{S_{12}^2}\left(\sigma_{12}^2+\sigma_{31}^2\right)\leq 1$$

Tensile Fiber Mode, $\sigma_{11} > 0$

$$\left(\frac{\sigma_{11}}{T_{11}}\right)^2 + \frac{1}{S_{12}^2}\left(\sigma_{12}^2 + \sigma_{31}^2\right) \leq 1$$

Compressive Fiber Mode, $\sigma_{11} < 0$

$$\left(\frac{\sigma_{11}}{C_{11}}\right)^2 \leq 1 \tag{10}$$

The Hashin criterion is thus composed of four separate modes of failure.

The Tsai-Wu criterion [3] combines all terms in (1) directly into a single mode of failure, and gives a seven property form involving the six properties in (5) plus one other, F_{12} below. The Tsai-Wu criterion is :

$$\left(\frac{1}{T_{11}} - \frac{1}{C_{11}}\right)\sigma_{11} + \left(\frac{1}{T_{22}} - \frac{1}{C_{22}}\right)(\sigma_{22} + \sigma_{33}) +$$

$$\frac{\sigma_{11}^2}{T_{11}C_{11}} + \frac{1}{T_{22}C_{22}}\left(\sigma_{22} + \sigma_{33}\right)^2 +$$

$$F_{12}\sigma_{11}(\sigma_{22} + \sigma_{33}) + \frac{1}{S_{23}^2}\left(\sigma_{23}^2 - \sigma_{22}\sigma_{33}\right) + \frac{1}{S_{12}^2}\left(\sigma_{12}^2 + \sigma_{31}^2\right) \leq 1 \tag{11}$$

The F_{12} parameter in (11) is usually called an interaction parameter. It must be specified by some auxiliary means. Often F_{12} is scaled differently by writing $2\,F_{12}$ rather than F_{12} in (11). Some published forms for the Tsai-Wu criterion also contain a second interaction parameter, F_{23}, in addition to the properties in (5), but that form is redundant and the form given here without F_{23} is preferable and consistent.

It is apparent that the present failure criteria (7) and (9) are much different from the Hashin and the Tsai-Wu forms. Later examples will reveal strong differences between all three failure criteria for carbon-polymer systems. The Tsai-Wu form does not decompose into separate fiber and matrix controlled modes, whereas the other two criteria do so decompose. This question of the possible decomposition or not of failure modes has always been of central importance to the field, and one which has generated strong positions and debates. This will be discussed further below.

The main difference between the present forms and the Hashin forms is that no assumptions are involved with the present forms whereas five particular assumptions were necessarily involved in the Hashin derivation. In addition, the Hashin method further decomposes the fiber and matrix controlled modes into sub-modes of either tensile or compressive nature. The isotropic material results of the previous section shows that this "sub-decomposition" is unnecessary and inappropriate.

Following the method given here (or that due to Hashin), under the condition of high anisotropy the failure characterization must decompose into the two separate failure criteria. On this basis, the Tsai-Wu form can only apply to moderately anisotropic systems, that is only for moderate departures from a state of isotropy. It cannot recover the limiting plane strain condition. The other two criteria do not apply under such moderately anisotropic conditions since they do not admit the limiting case of isotropy. They apply in the high anisotropy case typified by carbon-polymer systems. Thus the decomposed failure mode forms apply near one end of the anisotropy scale and recover the plane strain condition while the undecomposed form applies near the other end of the same scale and recovers the isotropy condition.

Considering the many composites failure criteria which have been proposed over time, the three general criteria discussed here stand out as having substantial derivations and developments. Of these three criteria, the present failure criterion, (7) and (9), has the most physically realistic basis for application to high performance fiber composite materials. An example of other criteria is that of Puck and colleagues, given by Puck and Schurmann [4]. Hinton, Kaddour, and Sodden [5] conducted an evaluation exercise for fiber composite failure criteria which gives a broad and helpful view of related matters.

Now return to the use and interpretation of the failure criteria (7) and (9). In order to give examples it is necessary to assign elastic and failure property values. Typical properties for carbon-epoxy systems are given by

$$
\begin{aligned}
E_{11} &= 150 \text{ GPa} \\
E_{22} &= 9 \text{ GPa} \\
\mu_{12} &= 6 \text{ GPa} \\
\mu_{23} &= 3 \text{ GPa} \\
\upsilon_{12} &= 1/3 \\
\upsilon_{23} &= 1/2
\end{aligned}
$$

and

$$
\begin{aligned}
T_{11} &= 2000 \text{ MPa} \\
C_{11} &= 1500 \text{ MPa} \\
T_{22} &= 40 \text{ MPa} \\
C_{22} &= 150 \text{ MPa} \\
S_{12} &= 80 \text{ MPa} \\
S_{23} &= 50 \text{ MPa}
\end{aligned}
$$

where the shear moduli and Poisson's ratios are included. It is seen that these typical properties conform to the high degree of anisotropy in both stiffness and strength.

The matrix controlled failure mode, (7), ordinarily involves curved failure envelopes such as the ellipses and parabolas of the isotropic case in the previous section. The most interesting cases are for stress states involving the fiber direction stress, σ_{11}, along with some of the other stress components, thus bringing in both failure modes, (7) and (9). Shown in Figs. 1 and 2 are the two stress states : σ_{11} vs. σ_{22} and σ_{11} vs. $\sigma_{22} = \sigma_{33}$, both cases with the other stresses as zero.

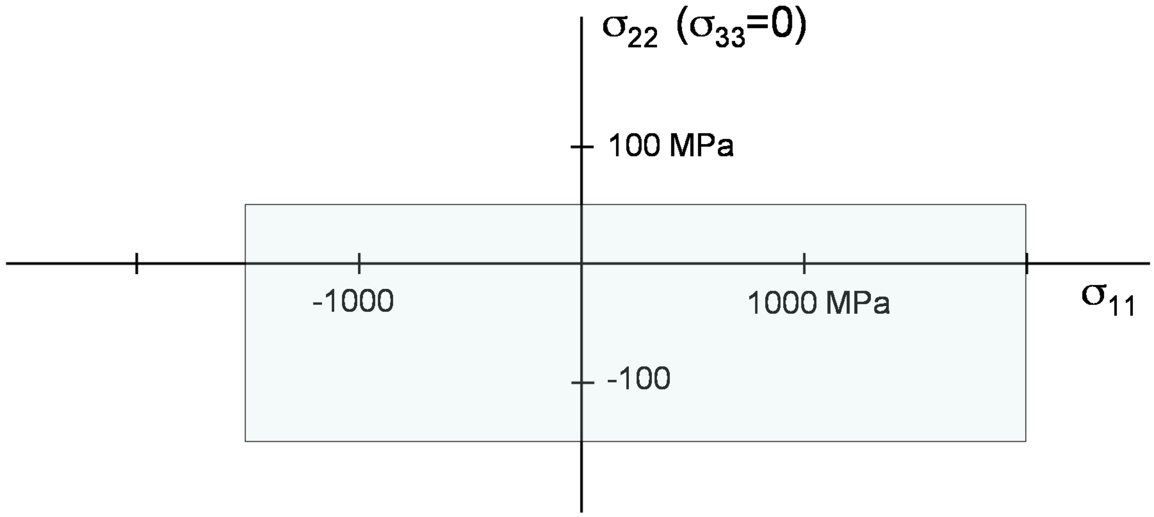

Fig. 1 2-D Stress State, Eqs. (7) and (9)

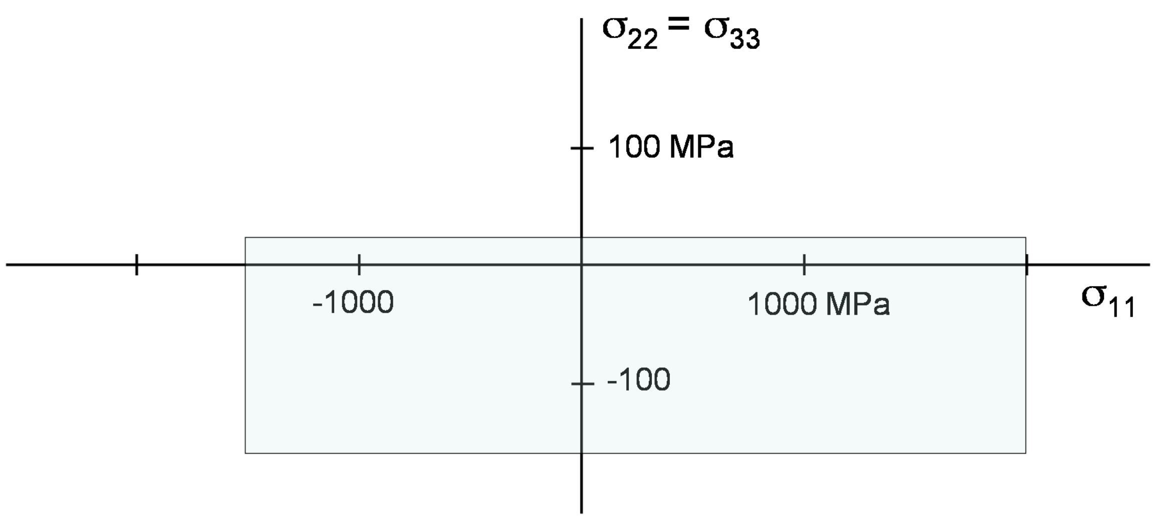

Fig. 2 3-D Stress State, Eqs. (7) and (9)

The 3-D case of Fig. 2 gives a transverse tensile value of about half that of the 2-D case in Fig. 1. Fig 2 gives a transverse compressive value slightly larger in magnitude that that of Fig. 1. The Hashin form (10) gives the same result for the case shown in Fig. 1 but does not give a lower limit for $\sigma_{22} = \sigma_{33}$ in the case of Fig. 2. The Tsai-Wu form (11) in both cases gives slender elliptical envelopes which at the maximum extend considerably beyond the fiber direction uniaxial strengths.

As a more involved example than those of Figs. 1 and 2 take the realistic stress state with σ_{11}=1500MPa (¾ of the uniaxial tensile strength) and then establish the failure envelope for σ_{22} vs. σ_{33}. The present criteria (7) and (9) gives an elliptical envelope. The Tsai-Wu criterion also gives an elliptical envelope, but it is considerably smaller and shifted from than that from (7). The Hashin criterion gives an open ended parabolic form. To be more specific in this example would require selecting a value for the F_{12} parameter in the Tsai-Wu form (11). Interested parties should work out cases such as these to show the differences between these three criteria, or any others. They all are fundamentally different.

The corners shown in Figs. 1 and 2 are the direct result of the decomposition into separate fiber controlled and matrix controlled modes of failure. This situation is completely parallel to that of the intersecting ductile and brittle failure modes of isotropic materials in the previous section. In physical reality, the corners would be expected to be rounded due to the small misalignment of testing specimens, and due to existing states of damage and specific inhomogeneities, as well as a host of other non-ideal effects. To some extent, this characteristic may also relate to the testing of lamina in isolation as opposed to the stabilized behavior of an in-situ lamina within a laminate. From this point of view, it is certainly best to use the forms directly from the basic failure criteria, (7) and (9), rather than to combine them with some artificial smoothing technique. Another problem with the regions around corners in failure surfaces is the extreme difficulty in generating reliable multi-axial test data. It is very much more difficult than that for the one dimensional strength tests of the basic properties in (5).

Fiber composites are rarely used in unidirectional form. Most commonly, lamina (or tows) are taken as the building blocks in laminates (or woven forms) composed of layers at various orientations. It is with laminates where the questions of corners in the failure surfaces are best considered and treated. Such topics will be taken up in the next section using the lamina level failure criteria (7) and (9) as the foundation and starting point

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